H32: Putting Numbers to Colour: CIE X Y Z

In 1931, the 8th session of the Commission Internationale de l' Eclairage (CIE) held in Cambridge, England, devised a system that provided positive tristimulus values for all visible colours. This was based on defining a new set of additive primaries X, Y, and Z based on the R G B set. The response of the eye is defined in terms of three new reference primaries X, Y and Z derived mathematically from the three real primary light sources R G and B. The X, Y, and Z stimuli are defined so that:-

The X Y Z tristimulus values of all real colours are positive.

The Y tristimulus value is proportional to the luminance.

The X Y Z values of the visible colours have the widest possible range of values.

Equal amounts of X, Y and Z has the same colour appearance as an equal energy white.

Imaginary primaries

The X, Y, and Z stimuli that satisfy these requirements cannot be physically created as light sources, for this reason they are termed ***imaginary primaries***.

The X stimulus represents a red more saturated than any spectral red.

The Y stimulus represents a green more saturated than any spectral green.

The Z stimulus represents a blue more saturated than any spectral blue.

Colour matching functions

The colour matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$, are a wavelength-by-wavelength representation of the amounts of the "imaginary" primary light sources, in T units, that are needed to match the colour sensation produced by a unit intensity of light with a narrow band of wavelengths centred on λ. The functions are plotted against wavelength in Figure 1.

Each of the curves in Figure 1 contains the same area so that an equal energy white will have

X = Y = Z

as well as

R = G = B.

The values shown in Figure 1 and used throughout this work, are taken from the ASTM document E308-85 (1985), "Standard methods for computing the colors of objects using the CIE system".

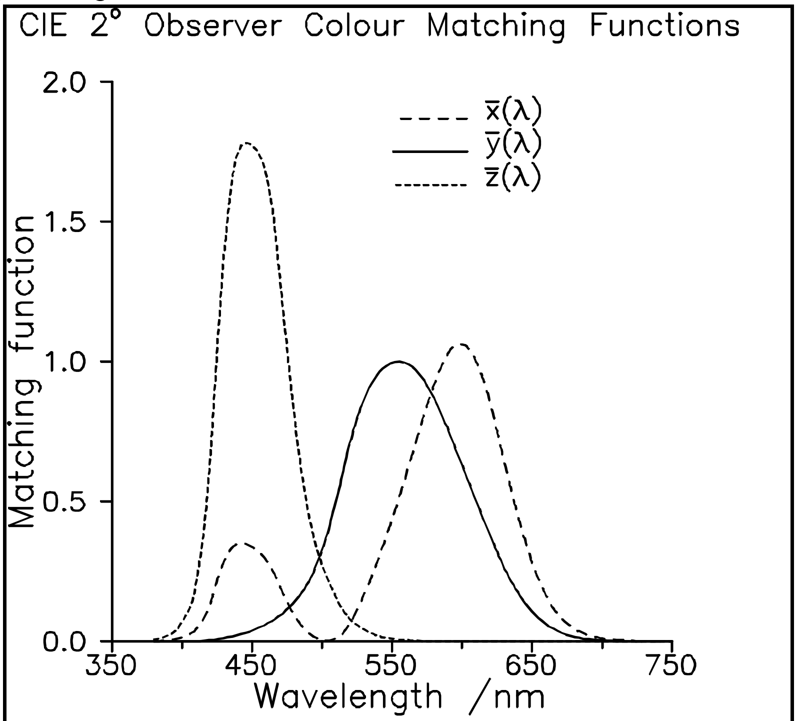

Figure 1: CIE 2°colour matching functions

The $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ functions are calculated from the experimentally determined $\bar{r}(\lambda)$, $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ values by using equations 1 to 3.

The constants in equation 2 were chosen so that the $\bar{y}(\lambda)$ values agree with the CIE luminosity function *V(λ)*, so that the value at 555 nm is 1.00, $\bar{y}(555)=1.000$.

Note, in absolute terms, 1 Watt of light intensity at 555 nm provides 683 Lumens of luminous flux.

Equation 1 $\bar{x}(\lambda) = 2.7689\times\bar{r}(\lambda) + 1.7517\times\bar{g}(\lambda) + 1.1302\times\bar{b}(\lambda)$

Equation 2 $\bar{y}(\lambda) = 1.0000\times\bar{r}(\lambda) + 4.5907\times\bar{g}(\lambda) + 0.0601\times\bar{b}(\lambda)$

Equation 3 $\bar{z}(\lambda) = 0.0000\times\bar{r}(\lambda) + 0.0565\times\bar{g}(\lambda) + 5.5943\times\bar{b}(\lambda)$

2° Standard Observer
The colour matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ were defined in 1931. They are known as the *2° observer colour matching functions* because the data was experimentally determined using a visual field subtending an angle of 2° at the eye. The light in this type of visual field just falls onto the foveal pit region of the retina.

10° Standard Observer
The accuracy of the 2° Observer data was questioned by a number of people and it was eventually recognised that the values were too low in the region 380 nm to 460 nm. In 1964 the CIE recommended a new standard observer based on measurements made with a visual field subtending an angle of 10° at the eye. The light in this type of visual field falls onto the foveal pit region of the retina and onto part of the surrounding regions. The colour matching functions and X Y Z values obtained using this data are denoted by the subscript 10, X_{10} , Y_{10} and Z_{10}.

Calculation of X Y Z tristimulus values
The following text illustrates the method of calculating the X, Y and Z values that specify the colour stimulus from a surface viewed under a particular illuminant.

Two additional parameters are need to be defined.

S(λ) The relative spectral power of the illuminant at wavelength λ.

R(λ) The fraction of incident light reflected by the surface at wavelength λ

As in the case of the R G B calculation, the spectrum of the light reflected by the surface is split into a number of thin bands, the colour sensation produced by each band is then individually matched with the X, Y and Z stimuli.

An example is given in the Figure 2 for the band of wavelengths centred on 620 nm.

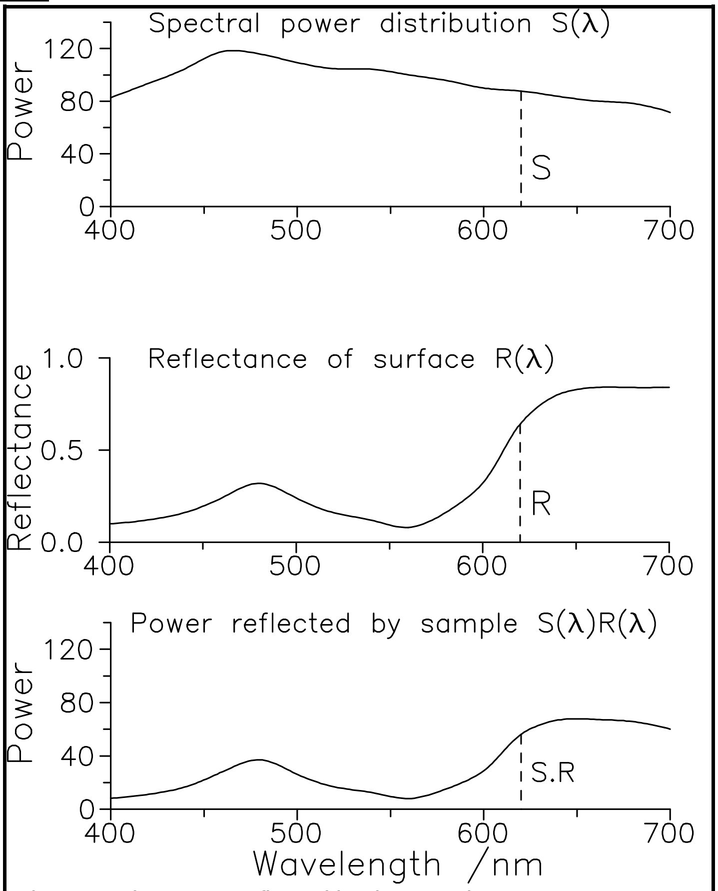

Figure 2: The power reflected by the sample at 620 nm

At 620 nm relative power of illumination is: S(620) = 87.70.

The reflectance of the sample is: R(620) = 0.6400

The relative amount of light at these wavelengths entering the eye is given by

$$I(\lambda) = S(\lambda).R(\lambda)$$

$$I(\lambda) = 87.70 \times 0.6400 = 56.13 \text{ units.}$$

The calibration values shown in Table 1 show that
to match a stimulus from unit intensity of light at 620 nm,

0.8544 T_X units of the X source (red),

0.3810 T_Y units of the Y source (green) and

0.0002 T_Z units of the Z source (blue) are needed.

To match the stimulus from an intensity of 56.13 then the amounts a, b and c of the X, Y and Z stimuli are needed, where:-

$$a = 56.13 \times 0.8544 = 47.96$$
$$b = 56.13 \times 0.3810 = 21.38$$
$$c = 56.13 \times 0.0002 = 0.01$$

So that $I(620) \equiv a + b + c$

A similar calculation can be made to match the contribution of each of the other thin wavelength bands in the spectrum of $I(\lambda)$. By Grassman's laws it follows that the total amounts of the X, Y and Z stimuli needed to match the colour appearance of the full spectrum entering the eye is the sum of the corresponding contributions.

$$I \equiv X + Y + Z$$

where

$$X = k[a(380) + a(400) + a(420)....], \text{ or } X = k \sum_{\lambda=380}^{\lambda=780} S(\lambda).R(\lambda).\bar{x}(\lambda)$$

$$Y = k[b(380) + b(400) + b(420)....], \text{ or } y = k \sum_{\lambda=380}^{\lambda=780} S(\lambda).R(\lambda).\bar{y}(\lambda)$$

$$Z = k[c(380) + c(400) + c(420)....], \text{ or } Z = k \sum_{\lambda=380}^{\lambda=780} S(\lambda).R(\lambda).\bar{z}(\lambda)$$

An example calculation is shown in Table 1 at the end of this section.

The ASTM 308-85 standard provides tables of data at 5, 10 and 20 nm spacing intervals. In general the narrower the spacing the more accurate will be the tristimulus values obtained from the summation. Summations over 5 or even 10 nm intervals are recommended for the most accurate work, however for many practical purposes summations over 20 nm intervals in the reduced wavelength range of 400 nm to 700 nm provide useful results.

Normalisation constant k

The normalisation constant has a special role in the calculation, its function is to ensure that the Y value of a perfectly reflecting white, $R(\lambda)=1$ at all wavelengths, has a value of 100. In other words the Y values are scaled to go from 100 for the perfect white surface to 0 for the perfect black surface.

$$k = \frac{100}{\sum_{\lambda=380}^{\lambda=780} S(\lambda).\bar{y}(\lambda)}$$

Chromaticity co-ordinates

The X Y and Z values provide a numerical description of the colour of a surface. These values are not easily related to the lightness, chroma and hue attributes of a colour. Progress can be made towards this type of description by using chromaticity co-ordinates.

The information describing the intensity of the colour sensation and the hue can be crudely separated out by considering the following process. If the intensity of light incident on the eye is increased then the values of X Y and Z will change in proportion. An observer would say that the luminosity had increased but the hue and saturation of the colour remain the same. The combined effect of hue and saturation is known as the chromaticity. The chromaticity of two colours is the same if the ratios of the amounts of X, Y and Z that match them are the same. Such colours only differ in luminosity. These ratios are termed chromaticity co-ordinates and are represented by the lower case letters x , y and z.

$$x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}, \quad z = \frac{Z}{X+Y+Z}$$

and of course $1 = x + y + z$

Chromaticity diagram

Only two of the three chromaticity co-ordinates are needed to specify the chromaticity of a colour, the value of the third can be calculated from the other two using equation 4.

The two chosen co-ordinates, usually x and y, can be plotted on ordinary graph paper, as shown in Figure 1. Such a plot is termed a chromaticity diagram and it is a useful way of representing the hue and saturation of a colour.

The usual arrangement for the chart is with x as the abscissa and y as the ordinate. The diagram also shows the line formed by the x, y values of monochromatic light at each wavelength in the spectrum. This line is called the *spectrum locus*. The spectrum locus plus the line joining the two ends (the purple line) enclose a region within which the co-ordinates of all real, visible colours must lie.

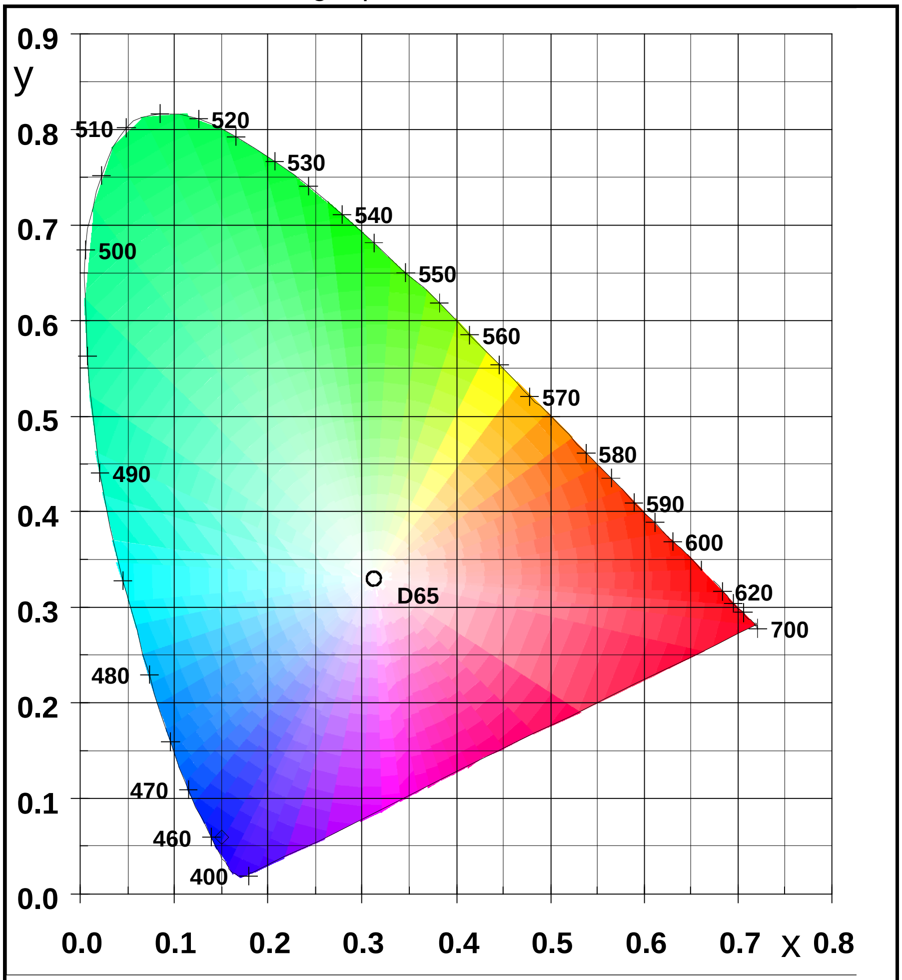

Figure 3: The CIE chromaticity chart

The diagram has a number of useful properties.

a) The co-ordinates of all colours that can be created by the additive mixture of light from two sources lie on the line joining the two points representing those sources.

b) The co-ordinates of colours produced by mixing light from three light sources all lie within the triangle joining the points representing those sources

c) The co-ordinates of the complement of any colour is found by drawing a line from the co-ordinates of that colour to the illuminant and then extending until it cuts the spectrum locus. The complementary colour will lie on the extended line joining the illuminant to the spectrum locus.

Dominant Wavelength and Purity

Dominant wavelength λ_d

Figure 4 shows the chromaticity co-ordinates of the following colours.

Sample M, x=0,1500, y=0.6000. Light source D65, x=0.3127, y=0.3290

The dominant wavelength (λ_d) of a colour is the wavelength of monochromatic light which, when added in the correct proportion to light from the Illuminant, produces a mixture that has the same hue and saturation as the given colour.

The points representing the sample (M) and the Illuminant (D65) are plotted on the chromaticity diagram, as shown in Figure 4. A line is drawn from D_{65} to M and extended until it cuts the spectrum locus at A. The wavelength at the point of intersection (A) is λ_d.

For sample (M), $\lambda_d = 514$ nm

Excitation purity ***P_e***

The excitation purity P_e of a colour is the relative proportion of light of the dominant wavelength in a mixture with light of the Illuminant that has the same hue and saturation as the given colour.

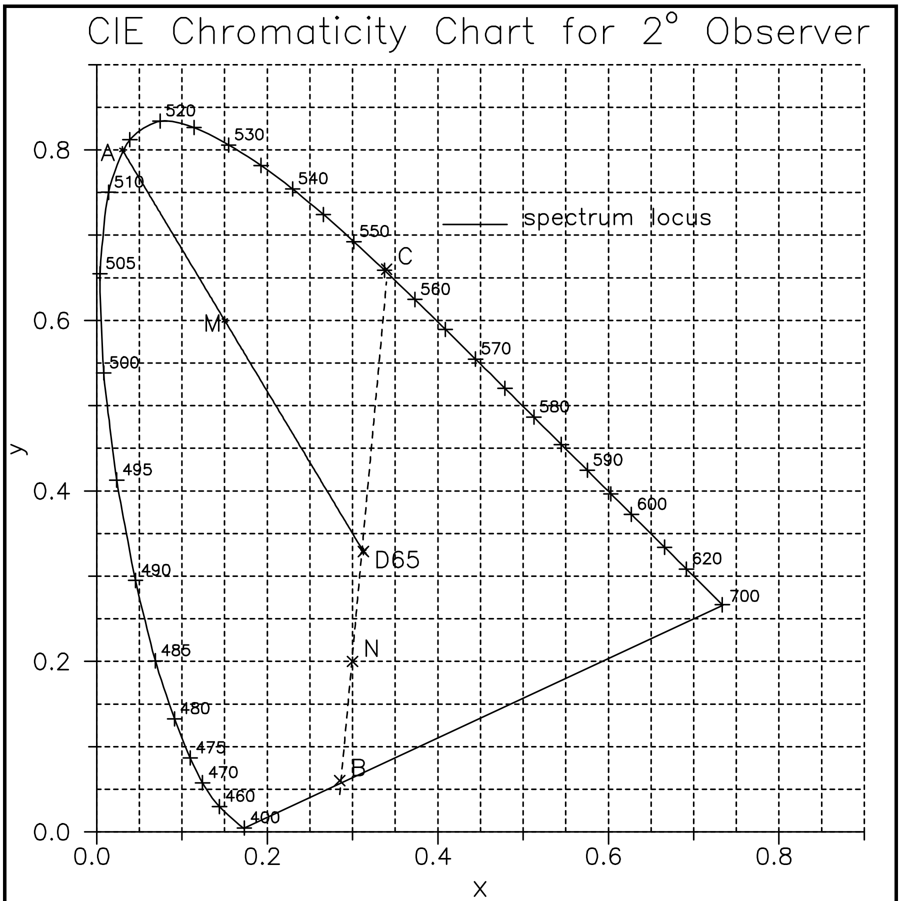

Figure 4: Determination of dominant wavelength and excitation purity

The excitation purity is the ratio of the distance (M to D65), to the distance (A to D65).

For sample (M), P_e = 0.583

In some cases, for example a sample with the colour co-ordinates N in the figure, the extrapolation cuts the purple line (B) rather than the spectrum locus. In these instances the complementary wavelength λ_c is quoted. This is found by extending the line backwards until it cuts the spectrum locus (C).

For sample (N), λ_c = 556 nm

The excitation purity is the ratio of the distance (N to D65), to the distance (B to D65).

For sample (N), P_e = 0.490

Limitations of the CIE X Y Z system

It is clear that while it is possible to define a colour by specifying

the values of X, Y and Z

and the illuminant (D65 or A or F7 for example)

and the observer (2° observer or 10° observer for example)

and the optical geometry of the colour measuring instrument (0/45, D/8 or T/6 for example)

The specification is by no means ideal for the following reasons.

It is difficult to interpret the hue, lightness and chroma of a colour from the X, Y and Z values or the chromaticity co-ordinates.

The degree of the visual difference in colour between two similar samples is not simply related to the numerical difference in the tristimulus values or to the numerical difference in the chromaticity co-ordinates.

Example calculation

Table 1: Spreadsheet calculation of X Y Z tristimulus values for the 2° observer, D65 Illumination calculated using data at 20 nm intervals from 400nm to 700nm

λ	**R(λ)**	**S(λ)**	**S×R**	**x(λ)**	**y(λ)**	**z(λ)**	**S×y**	**S×R×x**	**S×R×y**	**S×R×z**
400	0.1000	82.75	8.27	0.0143	0.0004	0.0679	0.03	0.12	0.00	0.56
420	0.1200	93.43	11.21	0.1344	0.0040	0.6459	0.37	1.51	0.04	7.24
440	0.1600	104.86	16.78	0.3483	0.0230	1.7471	2.41	5.84	0.39	29.31
460	0.2400	117.81	28.27	0.2908	0.0600	1.6692	7.07	8.22	1.70	47.20
480	0.3200	115.92	37.09	0.0956	0.1390	0.8130	16.11	3.55	5.16	30.16
500	0.2400	109.35	26.24	0.0049	0.3230	0.2720	35.32	0.13	8.48	7.14
520	0.1600	104.79	16.77	0.0633	0.7100	0.0782	74.40	1.06	11.90	1.31
540	0.1200	104.41	12.53	0.2904	0.9540	0.0203	99.61	3.64	11.95	0.25
560	0.0800	100.00	8.00	0.5945	0.9950	0.0039	99.50	4.76	7.96	0.03
580	0.1600	95.79	15.33	0.9163	0.8700	0.0017	83.34	14.04	13.33	0.03
600	0.3200	90.01	28.80	1.0622	0.6310	0.0008	56.80	30.59	18.17	0.02
620	0.6400	87.70	56.13	0.8544	0.3810	0.0002	33.41	47.96	21.38	0.01
640	0.8000	83.70	66.96	0.4479	0.1750	0.0000	14.65	29.99	11.72	0.00
660	0.8400	80.21	67.38	0.1649	0.0610	0.0000	4.89	11.11	4.11	0.00
680	0.8400	78.28	65.76	0.0468	0.0170	0.0000	1.33	3.08	1.12	0.00
700	0.8400	71.61	60.15	0.0114	0.0041	0.0000	0.29	0.69	0.25	0.00
							sum	**sum**	**sum**	**sum**
							529.54	166.28	117.67	123.26

Normalisation constant, k = 100/529.54 = 0.1888

X= 0.1888 x 166.28 = 31.40,

Y = 0.1888 x 117.67 = 22.22,

Z = 0.1888 x 123.26 = 23.28

H32: Putting Numbers to Colour: CIE X Y Z